4 Lecture 4 Notes: Introduction to Probability. Probability Rules. Independence and Conditional Probability. Bayes Theorem. Risk and Odds Ratio

Wrong is right. Thelonious Monk

4.1 Three Definitions of Probability

Definition 1: Relative Frequency Approximation

$$P(A) = \frac{\text{Number of times A occurred}}{\text{Number of times a trial was repeated}}$$

Example 1: P(A new born infant will live to see his or her first birthday in any given year and location)

Definition 2: Subjective Probability. P(A) is estimated by using previous knowledge.
Example 2: P(Candidate A wins an election)

Definition 3: Classical approach (requires equally likely outcomes). If event A can occur in s of n ways, then:

$$P(A) = \frac{\text{Number of ways A can occur}}{\text{Number of different simple events}}$$

Example 3: P(Getting a 4) when you roll a balanced die= 1/6

Law of Large Numbers: When a procedure is repeated again and again, the relative frequency probability tends to approach the actual probability.

4.2 Complement of Probability

Complement of a an event:
Notation: $\bar{A}$ or A^c
Consists of all outcomes in which event A does not occur.

Example 4: Event A is rolling a die and getting a 5
$P(\text{Getting a 5}) = P(A) = 1/6$
$P(\text{Not Getting a 5}) = P(\bar{A}) = 1 - 1/6 = 5/6$

4.3 Compound Event

A compound event: Is any event combining two or more simple events

Example 5: Getting an even number when rolling a die=$\{2, 4, 6\}$
Remember a simple event is just a single event, for example $\{3\}$

4.4 Rules of Probability

The Rules: For any event A

1. $0 \leq P(A) \leq 1$
2. If $P(A) = 1$, A always occurs and $P(\bar{A}) = 0$
3. If $P(A) = 0$, A never occurs and $P(\bar{A}) = 1$
4. $P(A) + P(\bar{A}) = 1 \rightarrow P(\bar{A}) = 1 - P(A)$

4.5 Union, Intersection, and Disjoint Events

Example 6: Say you have a die and a coin

1. Event A = Getting a head when a coin is tossed
2. Event B= Getting a 5 when a single die is rolled

Union of Events: The union of two sets is a new set that contains all of the elements that are in both sets.

$$\begin{aligned} P(A \cup B) &= P(A \text{ or } B) \\ &= P(\text{Event A occurs or event B occurs or they both occur}) \\ &= P(\text{Getting a head or getting a 5}) \end{aligned}$$

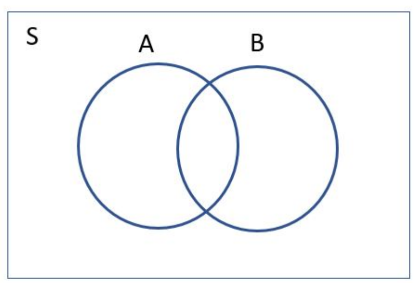
Intersection OF of Events: The intersection of two sets is a new set that contains the shared elements that are in both sets.

$$\begin{aligned} P(A \cap B) &= P(A \text{ and } B) \\ &= P(\text{Event A occurs and event B occurs simultaneously}) \\ &= P(\text{Getting a head and getting a 5}) \end{aligned}$$

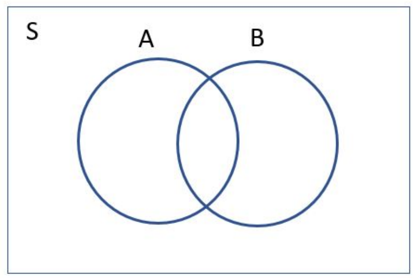
Disjoint Events (or mutually exclusive): They can not occur simultaneously

4.6 Venn Diagram

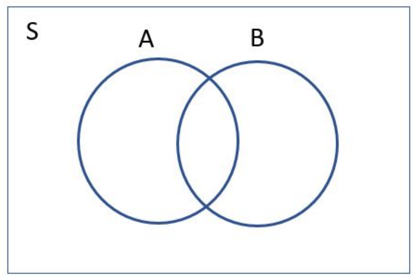
S: Sample Space
Union of A or B can be seen as $P(A \cup B)$



Intersection of A and B can seen as $P(A \cap B)$



Complement of A, $P(\bar{A})$



4.7 Probability Rules

Very important rule when calculating probabilities:

1. Addition Rule
2. Conditional Probability
3. Independence Events
4. Multiplication

Addition Rule
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Same thing $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

If the events are disjoints $P(A \cap B) = 0$
In this case $P(A \cup B) = P(A) + P(B)$

Example 7:
A die is rolled. What is the probability of getting a 1 or a 6?. $A = \{1\}$; $B = \{6\}$.
A and B are mutually exclusive.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) \\ &= 1/6 + 1/6 \\ &= 1/3 \end{aligned}$$

Conditional Probability: Probability of an event B given A occurs

$$\begin{aligned} P(B|A) &= P(\text{event B occurs given (after) event A has already occurred}) \\ &= P(B \text{ given } A) \end{aligned}$$

Independence Events Two events A and B are independent if the occurrence of one does not affect the probability of occurrence of the other. This means $P(B|A) = P(B)$ (Run to venn diagrams)

Example 8: When tossing a coin twice

$$P(\text{Head in a 2nd}|\text{Tail in a 1st}) = P(\text{Head in the 2nd}) = 1/2$$

Multiplication Rule Probability of an event A and B can be expressed as the probability of A multiplied by the the probability of B given A this can be expressed

$P(A \text{ and } B) = P(A \cap B) = P(A)P(B|A)$

If A and B are independents:

$P(A \cap B) = P(A \text{ and } B) = P(A)P(B) \quad (\text{because } P(B|A) = P(B))$

Example 9: A coin is tossed and a die is rolled.
P(Getting a head and Getting a 5) = P(H and 5)

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) \\ &= P(A)P(B) \quad A \text{ and } B \text{ are independent} \\ &= (\frac{1}{2})(\frac{1}{6}) \\ &= \frac{1}{12} \end{aligned}$$